A Theoretical Model of Public Response to the Homeland Security Advisory System

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Using a differential equation modeling approach, this paper explores the issue of public response to, and confidence in, anti-threat warnings. The effects of anti-threat warnings and their associated public confidence levels are modeled as a group of nonlinear differential equations. The analytical solutions of these nonlinear differential equations are derived to show how warning frequency and the duration of a warning affect public confidence, and how the effects of anti-threat warnings are constrained by the degree of public concern as the threat level changes. Phase plane analysis suggests that the number of warnings for a particular type of threat has a threshold level. Below this threshold, increasing the number of reliable warnings can improve the credibility and effectiveness of the warning system. However, once the number of warnings exceeds the threshold, the greater the number of warnings issued the less the public responds and the lower public confidence becomes. The resulting graphic representation is an easy-to-understand method for authorities to use to issue advisory warnings while maintaining the public's confidence in the system.

Keywords: Warning theory, nonlinear differential equation, public confidence, homeland security science

1. Introduction

Currently terrorist attacks are significant threats to U.S. homeland security. To protect the public and infrastructure, authorities usually issue a threat warning advisory to the public when there is a potential threat [2, 3, 8]. The warning advisory is issued through a five color–coded system (see Figure 1) which represents levels of risk related to a potential terror attack. Each threat level has a corresponding list of recommended actions that the public should take in order to reduce the likelihood or impact of a potential attack. Therefore, when a warning advisory is issued, authorities hope the public will follow the advisories listed on the DHS (Department of Homeland Security) Citizen Guidance on the Homeland Security Advisory System web page and take the recommended actions [1, 4]. For example, as of March 8, 2006 the country remains at an elevated risk, code yellow, for a terrorist attack. Given this threat level, the public is recommended to take the following twelve actions:

1) Develop a family emergency plan. Share it with family and friends, and practice the plan. Visit www.Ready.gov for help creating a plan.
2) Create an "Emergency Supply Kit" for your household.
3) Be informed. Visit www.Ready.gov or obtain a copy of "Preparing Makes Sense, Get Ready Now" by calling 1-800-BE-READY.
4) Know how to shelter-in-place and how to turn off utilities (power, gas, and water) to your home.
5) Examine volunteer opportunities in your community, such as Citizen Corps, Volunteers in Police Service, Neighborhood Watch or others, and donate your time.
6) Consider completing an American Red Cross first aid or CPR course, or Community Emergency Response Team (CERT) course.
7) Review stored disaster supplies and replace items that are outdated.
8) Be alert to suspicious activity and report it to proper authorities.
9) Ensure disaster supply kit is stocked and ready.
10) Check telephone numbers in family emergency plan and update as necessary.

Figure 1. Color-coded threat level system (Source: Homeland Security Advisory System, http://www.dhs.gov/dhspublic)

11) Develop alternate routes to/from work or school and practice them.
12) Continue to be alert for suspicious activity and report it to authorities.

If the code yellow alert (elevated) appears every day, it implies the warning advisory is issued every day. This suggests that the public needs to perform these twelve actions every day until the warning is off. The question that arises is whether these twelve actions are being reviewed and repeated as necessary by the public and are taken every day given the continuation of the code yellow alert. Recently we randomly interviewed 25 households living in the Chicago area and asked 1) whether they knew the current threat level of the warning advisory, and 2) whether they performed all the required recommendations corresponding to the current threat level. For question 1, nine households answered no, sixteen said they believed the alert was in code yellow since they did not hear of any change in the level of the alert. Regarding question 2, none of them performed the twelve recommendations listed above every day. When asked why, they mentioned they did follow the recommendations the first time the warning was issued. However, after several months, they became accustomed to the warning and felt that there was no difference whether they completed all recommendations or only some of them. Gradually, they stopped following the twelve recommendations even though the code yellow alert was still on.

Suppose the effect of a warning advisory is measured by examining whether the public responds to the issued warning by taking the recommended actions corresponding to a particular threat level. Then our small survey data seems to suggest that the continuous warnings may not generate the desired effect in terms of stimulating the public's response. We know that anti-threat warnings can help save lives and reduce the costs of potential disasters. However, warning about terrorist threats is different from the familiar warnings about severe weather. Warnings about severe weather will not change the occurrence of the weather event. That is, the severe weather will still occur no matter whether a warning is issued or not. But the warnings about terrorist threats may allow terrorist to alter targets, thereby escaping legal justice while still causing grave harm. Therefore, issuing an anti-threat warning may result in a change in the occurrence of a potential threat. Additionally, if the potential threat does not materialize each time the warning is issued and no public notice that the warning is over is given, the public may gradually lose attention and ignore these warnings resulting in a failure to perform the required recommendations. If this occurs frequently, it may gradually erode the credibility of the warning advisory system and public confidence [7]. In this paper, we develop differential equations to model the relationship between warning frequencies and their associated public response and confidence. Doing so will help us understand how to preserve public confidence in the warning advisory system while maintaining its effectiveness.

This paper is organized as follows. In section 2, we describe the problem setting and the model formulation which quantifies the interaction of the warning rate and the resulting level of public response and confidence. For our mathematical model, our philosophy is to examine the qualitative behavior of the model through phase analysis, and to investigate the quantitative behavior through finding an analytical solution of the model. Therefore, we analyze the qualitative behavior of the model using the perturbation method in section 3 and present analytic solution development in section 4. In section 5, we discuss major contributions, limitations of this research and avenues for future research.

2. Problem Setting and Model Formulation

According to DHS, the goal of issuing a warning for the authorities is to inform different levels of government agencies to take appropriate protective measures on one hand, and on the other hand to alert the public that there may be some type of threat to the United States and that the public needs to take informed actions [2, 3, 5]. In this paper, we focus on the public's responses. That is, we do not discuss how different levels of government agencies react to a threat warning. The term "response" used in this paper is defined as the public taking those recommended actions listed in DHS Citizen Guidance when a warning is issued. If the public does something other than these required actions, their response is counted as zero because the purpose of the Homeland Security Advisory System (HSAS) is to inform the public and suggest they perform specific tasks [2].

We define a threat event as an event that can cause the authorities to issue warning(s). When a decision to issue a warning is made, it normally covers the entire nation or a target region for a period of time. The target region is then in a threat state. For example, since its establishment in March 2002, the Homeland Security Advisory System (HSAS) national threat level has remained at elevated alert state, a code yellow warning has been "on," except for five periods during which the administration raised it to high alert where a code orange warning was issued. We also assume that the public consists of people who can understand the language used in the warning message after hearing or reading the warning and that the public is aware that each threat level has an associated list of recommended actions. Our research problem can therefore be described as follows.

Given a threat state e (i.e., $\forall e \in E$, where E denotes a set of threat states; a threat state corresponds to a threat level in HSAS) under which warnings would be issued for a target region r (i.e., $\forall r \in R$, where R denotes a set of target regions), we model how the warning rate influences its effectiveness in terms of public response and confidence in the target region r.

2.1 Model Formulation

For a particular threat state e, there are a corresponding maximum number of possible action items recommended by DHS. (See Appendix A for a detailed list of recommendations.) The public may incur time, labor, or monetary costs to perform each recommended action item. We use the term "task-load" to label such costs. Like many studies in economics, we can use money or time to measure task-load. Because the task-load required to complete every particular action item may be different for each individual, we let $P_{e,\max}$ denote the average task-load needed for the targeted population to complete all the action items recommended in state e by DHS. Note that the subscript "max" in $P_{e,\max}$ refers to the maximum number of possible action items listed for the corresponding threat state e. We define $p_e(t)$ as the amount of the task-load the public completes in response at time t in state e (i.e., equivalently the number of action items the public takes). When people understand and trust a warning, they are more likely to take these recommended protective actions. In other words, if people take action to respond, it must imply that they trust the warning information. From this point of view, $p_e(t)$ also indicates public confidence in the warning system.

Also let the duration of a warning be τ. The unit for τ is any convenient unit of time such as an hour, day, week, and so on. If the duration of a warning can be treated as continuously issuing the same type of warning signal, we can let $w_e(t) = t/\tau$, representing the number of warnings issued at time t in state e. Since the authorities control the warning frequency, the warning is an outside stimulus to the public. Theoretically, the authorities can issue endless warnings in state e. But intuitively the duration of warnings cannot last forever in state e, and the public's tolerance is limited. Since there exists a maximum possible duration time of warnings that the public can tolerate in state e, we let $W_{e,\max}$ represent this maximum value. In other words, it reflects the maximum number of warnings can be issued in state e without eroding the effectiveness of the warning system and public confidence.

Usually when a warning is issued, it should capture people's attention. If people do not realize there is a warning about an impending threat, they will not respond. Upon response, the amount of the task-load the public can actually complete (i.e., equivalently the number of action items the public can actually take) is also subject to the public's capability. Since humans have physical and mental limitations, a change in the amount of the task-load completed per unit time in state e (i.e., a change in the number of action items performed per unit time in state e) is proportional to the available capabilities of the targeted population. Let α be a positive proportionality constant indicating the degree to which the public perceives the warning information; then we can have the following differential equation:

$$\frac{dp_e}{dt} = \alpha\,[P_{e,\max} - p_e(t)]$$

where α is labeled as the attention coefficient.

Now we add to the model by assuming that the authorities may increase warning frequency to indicate the severity of the threat state or to reach those who ignored the earlier warnings. Then the average of the marginal increment in stimulating the public to perform recommended actions is also proportional to how many warnings that the public can tolerate in state e. That is,

$$\frac{\frac{dp_e}{dt}}{w_e} \propto [\,W_{e,\max} - w_e(t)\,]\,.$$

Combining the two equations above leads to

$$\frac{dp_e}{dt} = \alpha\,[P_{e,\max} - p_e(t)] + \beta\,[\,W_{e,\max} - w_e(t)\,]\,w_e(t)$$

where β is a positive constant.

In addition, to the public a warning is an outside stimulus. Therefore, when facing such a stimulus, people will decide whether the impending threat is relevant to them or if they are at risk. If relevant, they will check what action items they need to perform and whether they have the ability to complete the required task-load. Thus, a change in the number of warnings issued per unit time in state *e* is affected by the available capability the public possesses to complete the required task-load, as well as the extent to which the threat is geared to the public's immediate concern. That is, if people think they are not at risk, they may not respond. If we let the nonnegative constant γ be a measure of "perceived threat risk"—the extent to which the warning is geared to the public's immediate and relevant concerns (i.e., geographical proximity of the threat to nearby residents versus those living in another town), then we have another differential equation:

$$\frac{dw_e}{dt} = \gamma\,(\,P_{e,\max} - p_e(t)\,)\,.$$

Here, γ is also termed as the concern coefficient. Those at risk are more concerned than those who are not.

Combining all these equations above together, we have the following differential equation system capturing the relationship between warning rate and the public response and confidence.

$$\begin{cases} \dfrac{dp_e}{dt} = \alpha\,[P_{e,\max} - p_e(t)] + \beta\, w_e(t)[W_{e,\max} - w_e(t)] & (1) \\ \dfrac{dw_e}{dt} = \gamma\,[P_{e,\max} - p_e(t)] & (2) \end{cases}$$

subject to the initial conditions $w_e(0) = 0$, $p_e(0) = 0$; and nonnegative constants α, β, and γ.

We would like to check the way our model system behaves to understand the relationship between $w_e(t)$ and $p_e(t)$ with time *t*. Our general approach to achieving this goal is to examine the qualitative behavior of the model through phase analysis, and to investigate the quantitative behavior through finding an analytic solution of the model system.

3. Examining the Qualitative Behavior of the Model

Notice that our model system is nonlinear due to the presence of the $w_e^2(t)$ term in equation (1). It would be interesting to know whether the system behaves periodically. We then use the perturbation method to study the qualitative behavior of the system. This technique is especially useful to investigate a nonlinear autonomous system where it may be very difficult or perhaps even impossible to find analytical solutions [6].

3.1 Periodic or Nonperiodic Behavior?

Since $\dfrac{\partial}{\partial p_e}\left(\dfrac{dp_e}{dt}\right) + \dfrac{\partial}{\partial w_e}\left(\dfrac{dw_e}{dt}\right)$

$$= \frac{\partial}{\partial p_e}[\,\alpha\,(P_{e,\max} - p_e(t)) + \beta\, w_e(t)\,(W_{e,\max} - w_e(t))]$$

$$+ \frac{\partial}{\partial w_e}[\,\gamma\,(P_{e,\max} - p_e(t))]$$

$$= -\alpha < 0\ ,$$

therefore, we conclude that the system of equations (1) and (2) defines a family of non-closed and nonperiodic curves for $p_e(t)$ and $w_e(t) \geq 0$. This property is very important because it indicates that the public response and confidence, $p_e(t)$, does not behave periodically with $w_e(t)$, the number of warnings issued.

3.2 Finding Critical Points

To examine the qualitative behavior of the model system, we need to find the critical points of the model system. The critical points of the model system occur where $\dot{p}(t) = 0$ and $\dot{w}(t) = 0$.

Thus, setting the right-hand sides of both equations (1) and (2) equal to 0, we have

$$\begin{cases} \alpha\,[P_{e,\max} - p_e(t)] + \beta\, w_e(t)[W_{e,\max} - w_e(t)] = 0 & (3) \\ \gamma\,[P_{e,\max} - p_e(t)] = 0 & (4) \end{cases}.$$

Solving this pair of equations simultaneously for $p_e(t)$ and $w_e(t)$, we obtain two critical points $(p_e, w_e) = (P_{e,\max}, W_{e,\max})$ and $(p_e, w_e) = (P_{e,\max}, 0)$.

Point $(P_{e,\max}, 0)$ indicates that the public has completed the required task-load in state *e* when no warning is issued. This may imply that the targeted population is well trained and already at a heightened state of readiness. Thus no warning is necessary. However, currently the public obtains official threat alerts about terrorist attacks only from HSAS. That is, HSAS is the only means by which threat and advisory information is disseminated. From this point of view, even if the public is already at a heightened state of readiness, at least one warning needs to be issued for the purpose of notification. Otherwise, the public will not know of the existence of a potential impending threat. Due to this reason, we will not discuss the case of ($P_{e,\max}$, 0) in this paper.

Therefore we focus our phase plane analysis on point ($P_{e,\max}$, $W_{e,\max}$) as the single point of interest for the system (1) and (2).

3.3 Phase Plane of the System

Given the system of (1) and (2), we can determine the slope of the system's solution through any point (p_e , w_e). Graphically, we can draw a short line of the proper slope through each of many points (p_e , w_e) in the $p_e - w_e$ plane. This is called a direction field/map; see Figure 2. A solution curve of the system is then tangent to the direction line at each point through which the curve passes. Thus, the direction field gives a visual representation of what the family of possible solution curves to the system of (1) and (2) looks like. The construction of the direction field for a specific differential equation can be quite tedious to carry out. Below we give just a brief description of how we sketch the phase graph of the system in the first quadrant in the vicinity of the critical point ($P_{e,\max}$, $W_{e,\max}$) (because we are only interested in the area where $p_e \geq 0$ and $w_e \geq 0$). The $p_e - w_e$ plane can be divided into four areas based on the relationship between p_e and $P_{e,\max}$, as well as the relationship between w_e and $W_{e,\max}$; see Figure 2a.

3.3.1 Area 1 where $p_e < P_{e,\max}$ and $w_e < W_{e,\max}$

In this area, we have ($P_{e,\max} - p$) ≥ 0, ($W_{e,\max} - w$) ≥ 0, and $w \geq 0$. When Δt increases, we will get

$$\Delta p_e = \Delta t \left[\alpha (P_{e,\max} - p_e) + \beta w_e (W_{e,\max} - w_e) \right] \geq 0 .$$

So, Δp_e will increase with time *t* and

the direction of $\frac{\Delta p_e}{\Delta t}$ will be ⇒ .

Similarly, we can have $\Delta w_e = \Delta t \left[\gamma (P_{e,\max} - p_e) \right] \geq 0$, indicating that Δw_e also increases with time *t*. Therefore,

the direction of $\frac{\Delta w_e}{\Delta t}$ will be ⇑ .

Combining these two directions together, we will know that with time *t* increasing,

the combination direction of Δp_e and Δw_e will be [⇑ ⇗ ⇒]

as shown in a single arrow here (see Figure 2b).

3.3.2 Area 2 where $p_e < P_{e,\max}$, $w_e > W_{e,\max}$, $p_e > 0$, and $w_e > 0$

If the authorities continuously issue a warning in state *e*, then we have $\Delta t > 0$ and the warning increment $\Delta w_e > 0$. Therefore, we have

$$\frac{dw_e}{dt} = \frac{\Delta w_e}{\Delta t} > 0 .$$

Thus, with time *t* increasing,

the changing direction of $\frac{\Delta w_e}{\Delta t}$ is ⇑ .

Since

$$\frac{dw_e}{dt} > 0 ,$$

from equation (2) we can obtain that $\gamma (P_{e,\max} - p_e(t)) > 0$. This implies that $(P_{e,\max} - p_e(t)) > 0$ must exist because γ is positive. Suppose that $w_e(T) = W_{e,\max}$ at time $t = T$, we have $p_e(T) = P_{e,\max}$. This means that people completed the required task-load when $t = T$. Now when $t = T + \Delta t$, we have $w_e(T+\Delta t) = W_{e,\max} + \Delta w_e$, which is greater than zero. Since

$$\frac{dw_e}{dt} = \frac{w_e(T+\Delta t) - w_e(T)}{\Delta t} = \frac{\Delta w_e}{\Delta t} > 0 ,$$

from equation (2) we can infer that $\gamma \left[P_{e,\max} - p_e(T+\Delta t) \right] > 0$ holds. This implies that it must be $\left[P_{e,\max} - p_e(T+\Delta t) \right] > 0$ because γ is positive. Thus, we have

$$\frac{dp_e}{dt} = \frac{p_e(T+\Delta t) - p_e(T)}{\Delta t} = \frac{p_e(T+\Delta t) - P_{e,\max}}{\Delta t} < 0 .$$

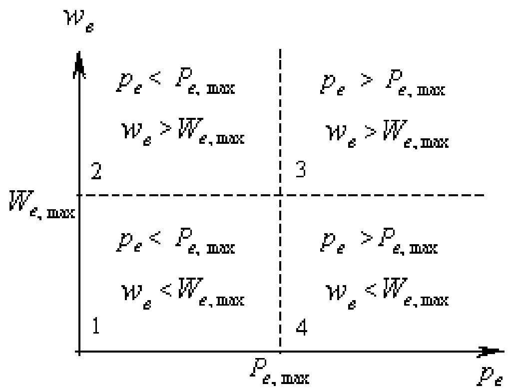
Therefore, with time increasing the changing direction of Δp_e is ⇐ .

The combination direction of Δp_e and Δw_e will be [⇑ ⇖ ⇐]

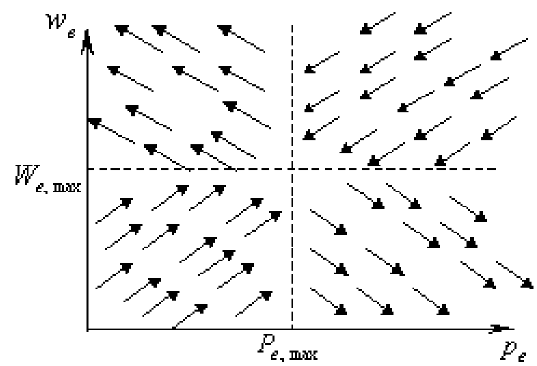
as shown in a single arrow (see Figure 2b).

Figure 2c displays the trajectories in the vicinity of the critical point ($P_{e,\max}$, $W_{e,\max}$), where arrows indicate the directions that solutions to $p_e(t)$ move as $w_e(t)$ increases.

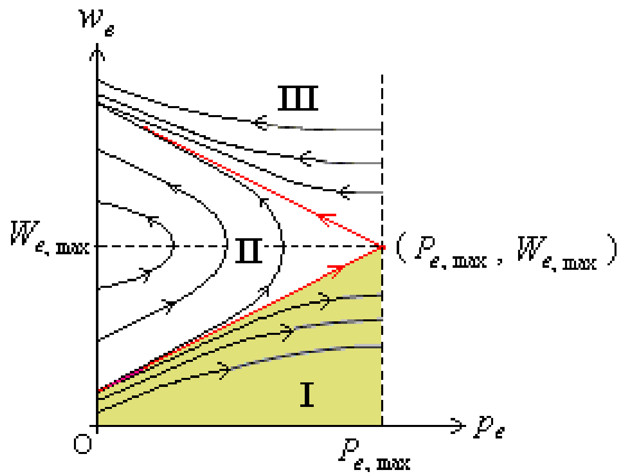
Since the trajectories are nonperiodic and non-closed curves, they show that, under the



(a) The First Quadrant: Four possible situations



(b) Short line segments and direction field



(c) Phase graph

Figure 2. Phase portrait of the system. The phase graph shows how p behaves with varying w with time increasing given a particular state. $p_e(t)$ increases when $w_e(t)$ increases in region I of Figure 2c. This indicates that increasing the number of warnings will stimulate the public to take the recommended actions, which in turn helps build public confidence in the warning system. But, when $w_e(t)$ exceeds $W_{e,\max}$, $p_e(t)$ will decrease with $w(t)$ (see region II). That is, once $w_e(t)$ > $W_{e,\max}$, $p_e(t)$ will change direction immediately and enter into region II. When this happens, the public's enthusiasm for response decreases as the number of alerts increases. This implies that the public response will gradually decrease to the point of inaction if too many similar alerts of the same type of threat are issued . Similarly, in region III of Figure 2c, increasing $w_e(t)$ will lead to the decrease of $p_e(t)$. In other words, if the number of warnings is above $W_{e,\max}$, the public response and confidence decreases with time t.

assumption of the system (i.e., $p_e(t)$ and $w_e(t) \geq 0$, $p_e(t) \leq P_{e,\max}$), $p_e(t)$ increases when $w_e(t)$ increases in region I of Figure 2c. This indicates that increasing the number of warnings will stimulate the public to take the recommended actions, which in turn helps build public confidence in the warning system. But, when $w_e(t)$ exceeds $W_{e,\max}$, $p_e(t)$ will decrease with $w(t)$ (see region II). That is, once $w_e(t)$ > $W_{e,\max}$, $p_e(t)$ will change direction immediately and enter into region II. When this happens, the public's enthusiasm for response decreases as the number of alerts increases. This implies that the public response will gradually decrease to the point of inaction if too many similar alerts of the same type of threat are issued and it will result in decreased public confidence in the warning system. Similarly, in region III of Figure 2c, increasing $w_e(t)$ will lead to the decrease of $p_e(t)$. In other words, if the number of warnings is above $W_{e,\max}$, the public response and confidence decreases with time t. The solution trajectories of the system support the evidence from our small sample survey which indicates that continuously issuing the same type of threat warnings may not generate the desired effect in terms of stimulating the public response.

The phase plane suggests that $W_{e,\max}$ for each event state plays an important role in public response and confidence. We are interested in how an appropriate $W_{e,\max}$ for each event state can be determined so that the proper number of warnings can be issued without eroding the effectiveness of the warning system or the level of public confidence in it. To answer this, we investigate the quantitative behavior of the model below.

4. Investigating the Quantitative Behavior of the Model

We can rewrite equations (1) and (2) as

$$\frac{dp_e(t)}{dw_e(t)} - \frac{\beta W_{e,\max} \times w_e(t) - \beta[w_e(t)]^2}{\gamma[P_{e,\max} - p_e(t)]} = \frac{\alpha}{\gamma} . \qquad (5)$$

Obviously, equation (5) is nonhomogeneous because the right-hand side is not equal to zero. Since the general solution to the nonhomogeneous equation is the sum of the complementary solution and any particular solution, we need to find the complementary solution to the associated homogeneous equation of equation (5).

The associated homogenous equation of equation (5) can be obtained by setting the left-hand side of equation (5) equal to zero. That is,

$$\frac{dp_e(t)}{dw_e(t)} - \frac{\beta W_{e,\max} \times w_e(t) - \beta[w_e(t)]^2}{\gamma[P_{e,\max} - p_e(t)]} = 0 . \qquad (6)$$

Extending equation (6) gives us

$$\gamma[P_{e,\max} - p_e(t)]dp_e(t) = [\beta W_{e,\max} \times w_e(t)]dw_e(t) - \beta[w_e(t)]^2\, dw_e(t) . \qquad (7)$$

Integrating both side of equation (7) from 0 to t yields

$$-\frac{\gamma}{2}[p_e(t)]^2 + \gamma P_{e,\max}[p_e(t)] + [\frac{\gamma\, p_e^2(0)}{2} - \gamma P_{e,\max} \times p_e(0)] = -\frac{\beta}{3}[w_e(t)]^3 + \frac{\beta W_{e,\max}}{2}[w_e(t)]^2 + [\frac{\beta\, w_e^3(0)}{3} - \frac{\beta W_{e,\max}}{2} \times w_e^2(0)] .$$

Substituting the initial conditions $w_e(0) = 0$ and $p_e(0) = 0$ into the above equation gives us

$$3\gamma[p_e(t)]^2 - 6\gamma P_{e,\max}[p_e(t)] = 2\beta[w_e(t)]^3 - 3\beta W_{e,\max}[w_e(t)]^2 . \qquad (8)$$

Solving for $p(t)$,

$$p_e(t) = P_{e,\max} \pm\sqrt{P_{e,\max}^2 + \frac{2\beta}{3\gamma}[w_e(t)]^3 - \frac{\beta}{\gamma} W_{e,\max}[w_e(t)]^2} .$$

Since $0 \le p_e(t) \le P_{e,\max}$, we take a "–" sign when determining to have a feasible solution. Therefore,

$$p_e(t) = P_{e,\max} - \sqrt{P_{e,\max}^2 + \frac{2\beta}{3\gamma}[w_e(t)]^3 - \frac{\beta}{\gamma} W_{e,\max}[w_e(t)]^2} . \qquad (9)$$

Equation (9) defines the solution trajectories of equation (6) in phase plane.

Let S_c denote equation (9). We borrow methods for solving constant-coefficient non-homogeneous linear equations (see Giordano and Weir [6]) and assume that this approach can be applied in a nonlinear situation. Then, the general solution to the system (5) will be the sum of the general solution of equation (6) and any particular solution of equations (1) and (2). Since the two critical points are also two particular solutions to the system of (1) and (2), a format of the general solution to our model system can be written as

$$\begin{bmatrix} p_e(t) \\ w_e(t) \end{bmatrix} = S_c + \begin{bmatrix} P_{e,\max} \\ W_{e,\max} \end{bmatrix} . \qquad (10)$$

Or

$$\begin{bmatrix} p_e(t) \\ w_e(t) \end{bmatrix} = S_c + \begin{bmatrix} P_{e,\max} \\ 0 \end{bmatrix} . \qquad (10')$$

Notice that S_c is given implicitly, and there is no general solution procedure for solving nonlinear equations. Thus, in appendix 2 we provide an approximation approach to find the closed form of the general solution to our model system.

Solving equations (9) and (10) in the vicinity of the critical point ($P_{e,\max}, W_{e,\max}$), that is, let $p_e(t) = P_{e,\max}$, we will get

$$P_{e,\max}^2 + \frac{2\beta}{3\gamma}[w_e(t)]^3 - \frac{\beta}{\gamma} W_{e,\max}[w_e(t)]^2 = 0 .$$

Because $w_e(t) = W_{e,\max}$ at the point ($P_{e,\max}, W_{e,\max}$), we substitute it into the above equation and obtain the following constraint relation:

$$W_{e,\max}^3 = \frac{3\gamma}{\beta} P_{e,\max}^2 . \qquad (11)$$

Theoretically, after the authorities decide the degree of risk (γ) regarding the threat event and the suggested recommendations they expect the public to take in state *e*, the authorities can use equation (11) to calculate the corresponding $W_{e,\max}$, the maximum number of allowable warnings that can be issued in state *e*.

5. Discussion and Conclusions

Effective anti-threat warnings can help save lives and reduce the costs of potential disasters. However, if the potential threat does not materialize each time the warning is issued and no public notice that the warning is over is given, if the warnings are issued too frequently, or if a warning lasts forever, the public may gradually ignore these warnings resulting in no performance of the required recommendations as suggested by our small survey data. If this occurs frequently, it may result in failure to respond in real emergencies, like the boy who cried wolf.

In this paper we construct a nonlinear differential equation model to understand how warning frequency impacts its effectiveness in terms of public response and confidence. We model the effectiveness of a warning advisory in terms of whether the warning (1) captures the public's attention, (2) is geared to the public's immediate and relevant concerns (i.e., degree of risk), and (3) prompts people to follow the advisories listed in the guidelines corresponding to each particular threat level and take the suggested actions. A warning should stimulate people to take informed actions. So when a warning is issued, if people take action to perform the recommendations suggested by DHS, it must imply that they trust the warning. The qualitative behavior of our model system predicts that the number of warnings for a particular threat state issued without eroding the effectiveness of the warning system and public confidence has a threshold level. Below this threshold value, increasing the number of reliable warnings can improve the credibility and effectiveness of the warning system. However, once the number of warnings exceeds the threshold, the greater the number of warnings issued the less the public responds and the lower public confidence becomes. When this happens, the impact of warnings will have the opposite effect of the one expected. If too many alerts in the same threat state are issued or if a single warning is repeated for a long time, the public will gradually lose attention and ignore them. Our model's prediction is consistent with our small survey result.

In addition, a formula for calculating a threshold of warnings for each threat state is derived. It suggests that the threshold value is constrained by the perceived risk of threat and the public's capability to respond. Because threats do not always materialize, it is expected that the authorities give detailed explanations for any warning for which the threat proves false in order to prevent the development of public distrust. This increases flexibility in the number of effective warnings the authorities can issue.

Note that our model is formulated based on the setting described in section 2—given a state *e* (i.e., a threat level) under which warnings are issued. That is, the model is threat-state specific. For example, in an elevated risk state, the authorities issue a code yellow warning. However, when in a high risk level, a code orange warning is issued. So, code yellow and code orange represent two warning signals and are in two different states: the elevated risk state and the high risk state, respectively. We use a subscript *e* to indicate different individual states (or threat levels). Therefore, *e* can be "elevated risk state," "high risk state," "severe risk state," and so on.

As in the study of more general dynamic systems (i.e., in economic, biological, or electrical areas, and so on), we use a continuous system approach (i.e., the system is in the form of a set of differential equations) to study human social behavior in this paper. However, a continuous social behavior sometimes may be interrupted by a sudden catastrophic non-continuous event. If this occurs, our model system would undergo a sudden switch from one equilibrium state to another, as shown in Figure 3. Here, an outside force may cause the system to make such a sudden jump. For example, suppose that our model system is in a particular state (e.g., the threat level is low risk). Now if the warning issuing authorities suddenly raise the threat level from low risk to elevated risk due to the events of September 11 (where code yellow is issued), the model system would switch to a new state labeled as elevated risk or code yellow. In this new state (i.e., *e* = elevated risk state), the system of equations (1) and (2) displays how the public response behaves with varying warning rates. Here, the authorities' raised threat level, as an outside force to the public, makes the system jump from the low risk state to the elevated risk state. Since the public does not know how the authorities determine the level of the risk state, if we assume such a determination is based on a number of factors unknown to the public but known by the authorities, we can use a function $G(v_1, v_2, \cdots, v_m)$ to represent such a decision, where $v_1, v_2, \cdots, v_m$ are various factors. Then we can conclude that our model system would have a sudden state jump/switch when the function *G* is suddenly added to the system.

In this paper we focus on public response and confidence. Future research could be extended to study how government agencies determine: (1) the change of threat state, that is, the explicit quantitative

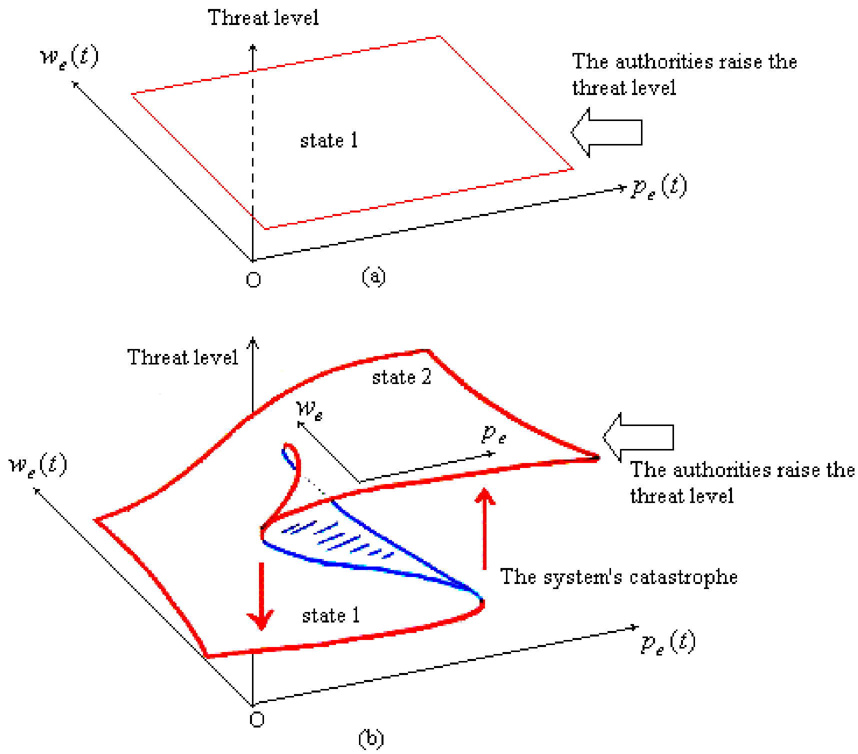


Figure 3. The system's switching behavior due to a sudden catastrophe. Figure 3a shows that the system is in a particular state (e.g., the threat level is state 1). Now, the authorities suddenly raise the threat level to state 2, this makes the system undergo a sudden switch from state 1 to state 2 as shown in Figure 3b.

expression of the function $G(v_1, v_2, \cdots, v_m)$ as suggested in [5]; (2) what type of action items and how many action items the public in different target areas need to perform in each threat state since people living in different geographic areas may have different requirements; and (3) when a public notice that a warning is over should be issued to improve the design of the homeland security advisory system itself. Our model also has some limitations. The attention coefficient, α, is treated as a constant in this paper. In the real world, it may vary with time t because the human body and mind may become fatigued over time. Thus, the attention coefficient α can be modeled as a function of time. Also constants β and γ may be determined through a sampling survey or historical data. Our research could provide theoretical foundations for such empirical studies.

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Appendix 1. DHS Citizen Guidance

(Available online at http://www.dhs.gov/dhspublic/.)

READY.GOV
From The U.S. Department of Homeland Security.

Citizen Guidance on the Homeland Security Advisory System

Risk of Attack	Recommended Actions for Citizens
GREEN Low Risk	Develop a family emergency plan. Share it with family and friends, and practice the plan. Visit www.Ready.gov for help creating a plan. Create an "Emergency Supply Kit" for your household. Be informed. Visit www.Ready.gov or obtain a copy of "Preparing Makes Sense, Get Ready Now" by calling 1-800-BE-READY. Know how to shelter-in-place and how to turn off utilities (power, gas, and water) to your home. Examine volunteer opportunities in your community, such as Citizen Corps, Volunteers in Police Service, Neighborhood Watch or others, and donate your time. Consider completing an American Red Cross first aid or CPR course , or Community Emergency Response Team (CERT) course .
BLUE Guarded Risk	*Complete recommended steps at level green.* Review stored disaster supplies and replace items that are outdated. Be alert to suspicious activity and report it to proper authorities.
YELLOW Elevated Risk	*Complete recommended steps at levels green and blue.* Ensure disaster supply kit is stocked and ready. Check telephone numbers in family emergency plan and update as necessary. Develop alternate routes to/from work or school and practice them. Continue to be alert for suspicious activity and report it to authorities.
ORANGE High Risk	*Complete recommended steps at lower levels.* Exercise caution when traveling, pay attention to travel advisorie. Review your family emergency plan and make sure all family members know what to do. Be Patient. Expect some delays, baggage searches and restrictions at public buildings. Check on neighbors or others that might need assistance in an emergency.
RED Severe Risk	*Complete all recommended actions at lower levels.* Listen to local emergency management officials. Stay tuned to TV or radio for current information/instructions. Be prepared to shelter-in-place or evacuate, as instructed. Expect traffic delays and restrictions. Provide volunteer services only as requested. Contact your school/business to determine status of work day.

**Developed with input from the American Red Cross.*

Appendix 2. A Form of the General Solution Using Linear Approximation

Since the critical point ($P_{e,\max}, W_{e,\max}$) is a particular solution to the system of (1) and (2) and it is the one that we are interested in this paper, let $x(t) = p_e(t) - P_{e,\max}$, and $y(t) = w_e(t) - W_{e,\max}$. Doing this gives

$$\frac{dx}{dt} = \frac{\partial x}{\partial p_e}\cdot\frac{dp_e}{dt} = \frac{\partial}{\partial p_e}[\,p_e(t) - P_{e,\max}\,]\cdot\frac{dp_e}{dt}$$

$$= \alpha\,[-x(t)] + \beta[\,y(t) + W_{e,\max}\,]\cdot[-y(t)]$$

and

$$\frac{dy}{dt} = \frac{\partial y}{\partial w_e}\cdot\frac{dw_e}{dt} = \frac{\partial}{\partial w_e}[\,w_e(t) - W_{e,\max}\,]\cdot\gamma[\,P_{e,\max} - p_e(t)]$$

$$= \gamma[-x(t)]\ .$$

Rewriting them, we have

$$\dot{x}(t) + \alpha\,x(t) + \beta W_{e,\max}\,y(t) = -\beta\,y^2(t) \qquad \text{(A1)}$$

and

$$\dot{y}(t) + \gamma\,x(t) = 0\,. \qquad \text{(A2)}$$

Since (A1) is a nonlinear differential equation, we approximate it with its associated homogenous linear equation. Now the system of (A1) and (A2) becomes

$$\dot{x}(t) \simeq -\alpha\,x(t) - \beta W_{\max}\,y(t)$$

$$\dot{y}(t) = -\gamma\,x(t)\,.$$

That is,

$$\begin{bmatrix}\dot{x}(t)\\ \dot{y}(t)\end{bmatrix} = \begin{bmatrix}-\alpha & -\beta W_{\max}\\ -\gamma & 0\end{bmatrix}\cdot\begin{bmatrix}x(t)\\ y(t)\end{bmatrix}. \qquad \text{(A3)}$$

The eigenvalues for (A3) are

$$\lambda_{1,2} = -\frac{\alpha}{2} \pm \sqrt{(\frac{\alpha}{2})^2 + \beta\gamma W_{e,\max}}\ . \qquad \text{(A4)}$$

Then the general solution to (A3) is

$$\begin{bmatrix}x(t)\\ y(t)\end{bmatrix} = c_1\begin{bmatrix}a_1\\ b_1\end{bmatrix}e^{\lambda_1 t} + c_2\begin{bmatrix}a_2\\ b_2\end{bmatrix}e^{\lambda_2 t}\ ;$$

where a_1, b_1, a_2, b_2, c_1, and c_2 are constants.

Therefore, the general solution to the linear approximated system of the original (1) and (2) is then,

$$\begin{bmatrix}p_e(t)\\ w_e(t)\end{bmatrix} = \begin{bmatrix}k_1\\ k_2\end{bmatrix}e^{\lambda_1 t} + \begin{bmatrix}m_1\\ m_2\end{bmatrix}e^{\lambda_2 t} + \begin{bmatrix}P_{e,\max}\\ W_{e,\max}\end{bmatrix}$$

where $\begin{bmatrix}k_1\\ k_2\end{bmatrix} = \begin{bmatrix}c_1\,a_1\\ c_1\,b_1\end{bmatrix}$ and $\begin{bmatrix}m_1\\ m_2\end{bmatrix} = \begin{bmatrix}c_2\,a_2\\ c_2\,b_2\end{bmatrix}$.

Acknowledgements

The author thanks the editor and three anonymous referees for valuable comments.

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