Modeling the Lanchester Laws with System Dynamics

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This study develops a framework using the Lanchester Laws in a hybrid system dynamics model. Two models of the Lanchester Laws are compared. The first is a traditional discrete event simulation and the other is a system dynamics model. System dynamics models are able to represent qualitative aspects of combat that are difficult to capture in a discrete event simulation. This is directly applicable to modeling combat since some key drivers within the expected environment are difficult to model. Additionally, the system dynamics construct allows for the modeling of continuous events and incorporating system feedback. Both of these properties are favored when modeling combat or asymmetrical operations. Results from the discrete event simulation and the system dynamics model are analyzed and contrasted.

Keywords: System dynamics, combat modeling, Lanchester Laws, attrition

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1. **Introduction**

System dynamics models are able to represent qualitative aspects of combat that may be difficult to capture in a discrete event simulation. Additionally, the system dynamics construct allows for the modeling of continuous events and incorporating system feedback. This study develops a systems dynamics framework of the Lanchester Laws with events. Two models of the Lanchester Laws are presented. The first is a traditional discrete event simulation which is offered as a baseline for comparison; the other is a system dynamics implementation of the Lanchester Laws. Results from the discrete event simulation and the system dynamics model are analyzed, compared and contrasted. This paper begins with a brief overview of discrete event simulations and system dynamics modeling. A discussion of the Lanchester Laws is next presented. The study then provides examples of how the Lanchester Laws can be modeled using both discrete event simulation and system dynamics. Comparisons of the modeling techniques are presented through the implementation of an excursion (compared to a baseline) where reserve forces are implemented. The suitability of the system dynamics framework in some settings is further demonstrated with an excursion where the forces are fatigued. The paper concludes with a summary of results and suggestions for future research.

2. Discrete Event Simulation

Law and Kelton define discrete event simulation as "the modeling of a system as it evolves over time by a representation in which the state variables change instantaneously at separate points in time [1]." Many of today's combat models can be characterized as discrete event simulations. These include THUNDER, STORM, CEM, TACWAR and JWARS, among others. (THUNDER and STORM are the leading campaign models used by the US Air Force. CEM is used by the US Army Center for Army Analysis. TACWAR is the official campaign model of the Joint Staff. JWARS is a next generation campaign model under development by JFCOM).

Kelton, Sadowski and Sadowski [2] suggest nine elements of discrete event simulations: Entities, Attributes, Resources, Queues, (Global) Variables, Statistical Accumulators, Events, Simulation Clock, and Starting and Stopping. An entity is the first common piece of a discrete event simulation. The entity is created by the user or the model. As the entity moves through the system, resources are expended. The waiting for, and consumption of, these resources will create queues within the system. Additionally, variables will be globally changed within the system.

Statistical accumulators are used to measure the status of the attributes, variables, and queues. Events are the time that a change to variables, entities, and attributes occurs within the simulation. The time of the event is tracked by the simulation clock. This is not a continuous clock but a time sequential list of events. Finally, the simulation must have starting and stopping conditions.

Banks [3] lists thirteen advantages of discrete event simulations. Ten of these thirteen advantages are inherit in modeling a system, regardless of the tool being used. However, three of Banks' advantages are more directly related to discrete event simulation as opposed to some other methodologies. First, discrete event simulations can compress and expand time. The time clock within a simulation can be altered to investigate specific events in great detail or increased so the impact over days/weeks/months/years can be investigated.

Next, discrete event simulations can be used to identify constraints. Simulations can reveal the bottlenecks within complex systems. This can assist in revealing the causes of delays. Discrete event simulation modeling and data collection of queues is specifically significant.

Finally, discrete event simulation can help visualize the interactions within the analysis. Most modern simulations have animation features which allow the users to visualize the events but generally require smaller time slices. The focus of a discrete event simulation, however, is generally the actions at the event instead of the entirety of the system.

As suggested earlier, most combat models have traditionally been implemented as discrete event simulations. The advantages and flexibility of discrete event simulations have allowed an array of applications of combat to be modeled by this approach from the high fidelity engineering-level models to the lower-fidelity campaign-level models.

3. System Dynamics

Some of the qualitative aspects of combat may be difficult to capture in a discrete event simulation. System dynamics provides an option to discrete event simulation which may provide additional insights to the analyst and decision maker and, where appropriate, possibly a better representation of decision environment. In this section, system dynamics is introduced and the concepts of feedback and continuous flow, which are critical to system dynamics, are described.

Forrester defines a system as a “grouping of parts that operate together for a common purpose [4].” This grouping can include the parts, people, and/or resources required of the system. A system of combat might include the equipment, troops, leaders, and supplies of both sides of the battle. Sweetser states that “dynamics refers to change over time. System dynamics is, therefore, a methodology used to understand how systems change over time [5].” The system dynamics society defines system dynamics as the “methodology for studying and managing complex feedback systems [6].”

Forrester discusses two types of systems; ‘open’ and ‘feedback’ systems [4]. An open system takes inputs and produces an output that does not influence future inputs. A feedback system, or closed system, uses the outputs of one cycle to effect inputs for future cycles of decisions. Thus, a system with feedback is preferred when decisions are made by decision makers which rely on the past performance or can affect functions integral to future decisions. This is clearly a trait of combat. Additionally, the current decision impacts future locations of troops and material and therefore future options and decisions.

A system dynamics model changes in a transient nature rather than at an event. This implies that the system is continuous in both states and variables. Thus, the rate of effectiveness is continuously changing over time rather than held at a constant until the next event which will change the rate occurs. A final attribute of system dynamics models is their non-linear nature; the model can capture effects which are more than additive.

The concepts of levels and rates drive system dynamics models. When a system is at rest, the levels are the amount of materials and supplies that are present [7]. This might be viewed in a combat model as the number of troops, equipment, or material present at any point during the model. The rates are the instantaneous flows of materials and items rather than time stepped increments in discrete event simulations [7]. The rates of flow can depend upon levels determined within the model. Additionally, information about the levels must be communicated to adjust the “valve” which controls the rate of flows within the model. Figure 1 is a representation of a system dynamics model with positive feedback presented by Kirkwood [8].

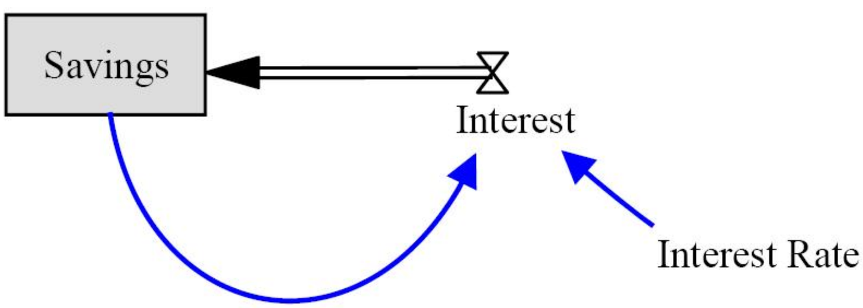

Figure 1: System Dynamics Model of Savings

The Savings is a level and is indicated by a square container. Interest is a decision function that is represented by a valve. The valve controls the rate of flow into Savings from Interest which is represented as the double arrow. Finally, the Interest is dependent upon the current level of Savings and the Interest Rate. This dependency is identified by the single arrows. The Interest causes a rate to change the amount of the Savings. Assuming an initial Savings level of $100 and an Interest Rate of 5% per year, the positive feedback and growth of the system can be observed.

System dynamics models can easily be adjusted by the decision maker to test new and different scenarios [5]. This includes adding new linkages and feedback loops to the system. Additionally, system dynamics models are designed to model continuous processes rather than discrete events [5]. The flow of battle can certainly be seen as a continuous event.

System dynamics models are able to represent "softer" qualitative aspects, such as behavior. These aspects can be difficult to model in a discrete event simulation. Coyle [9] states, "system dynamics, however and rightly, is strategic in orientation and it is often seen as necessary to introduce 'soft' variables." Examples of these softer variables include: consumer satisfaction, chaos, suffering, and other human activities [10]. Forrester's model of *Urban Dynamics* [11] included several social issues within a city's development, while in *World Dynamics,* Forrester [12] integrated social, economic, and environmental attributes to build models of a much larger scale. For further information regarding the use of system dynamics to model the qualitative effects and the softer aspects see Aracil [10], Coyle [9], and Forrester [13], among others. The application of qualitative aspects is pertinent to combat since such drivers as morale, training, and leadership are all qualitative concepts which historically have been difficult to model.

Another feature of system dynamics models, similar to complex adaptive systems, is these models are focused on the internal dynamics of the individual system. The evolution of the system is more important than the specific event that precipitated the solution [5]. This concept is also valued in combat modeling, as the solution or victory may be less important than the path to achieve the victory in a modeling analysis.

A final advantage of system dynamics models is the choice of time step does not impact the results. This allows the modeler to focus on the interactions within the model and not the size of the time step to get the required level of detail.

System dynamics use of both positive and negative feedback and specific rates can add complexity to the model. Just as with classic Lanchester models and discrete simulations, it may be a challenging task to properly fit the relations and parameters of a systems dynamic model. As with any model, it must be verified and validated to provide confidence in the model. Forrester and Senge suggest a three phase process to build confidence in system dynamics models. The phases validate the model structure, model behavior, and policy implications. See Forrester and Senge [14], Sterman [15], or Coyle and Exelby [16], among others for applications, clarifications and insights when verifying and validating system dynamics models. Effort will need to be expended to fit specific models and verify and validate systems dynamic models.

4. Lanchester Laws

Beginning in September 4, 1914, Lanchester published a series of sixteen weekly articles entitled "Aircraft in Warfare: The Dawn of the Fourth Arm." The fifth and sixth articles, printed on October 2nd and 9th are the foundations of the Lanchester Laws. A brief summary is provided in this section. MacKay [17] provides an excellent mathematical summary of the Lanchester Laws.

Lanchester begins by proposing that the ancient battle was comprised of several individual duels. Lanchester states that in an ancient battle there is "little or no importance" between a battle of 1000 blue forces versus 1000 red forces and 500 blue forces versus 500 red forces [18]. These two battles will result in the same victorious side depending only on the individual fighting value and not numerical strength of the forces. Although the equations were never directly listed, Taylor and others suggest that Lanchester could mathematically represent his statements of the linear ancient battle as Equation 1 (adapted from [19]).

$$\frac{dR}{dB} = E = \frac{c}{k} \quad (1)$$

where, b and r are the number of blue and red forces engaged, c and k are the individual fighting values for the red and blue forces (better known as the Red / Blue attrition coefficients), and E is the exchange ratio. Equation 1 can be integrated and rewritten as the Lanchester Linear Law, Equation 2 (adapted from [19]).

$$k[R_0 - R(t)] = c[B_0 - B(t)] \quad (2)$$

Equation 2 assumes, R_0 and B_0 are the initial sizes of the red and blue forces, and $B(t)$ and $R(t)$ are the number of loses for blue and red forces, respectively, at time t.

The Linear Law can also be demonstrated as the Area Fire Equation (Equation 3):

$$\frac{dB}{dt} = -R \times B \times c$$
$$\frac{dR}{dt} = -R \times B \times k \quad (3)$$

The Area Fire Equations have been used to describe the effectiveness of attritting an enemy's army by shooting into any area versus directly targeting an individual. This characterizes both the impacts of artillery and shooting in the direction of a non-tracked target.

Taylor provides the following example as an application of the Lanchester Linear Law. For this example, let there initially be one hundred red forces (R_0 = 100), and let $k = c$, thus the exchange ratio is one (E=1). Additionally, let the initial number of blue forces (B_0) increase from 100 to 300 in increments of 50. Solving for the size of blue's forces surviving ($B_f = B_0 - B(t)$) when all red forces are annihilated ($R(t) = 100$) provides the following results given in Table 1.

Table 1: Ancient Warfare Example (adapted from [19])

B_0	100	150	200	250	300
B_f	0	50	100	150	200
Blue losses	100	100	100	100	100

* Fighting continues until $R_f = 0$

Lanchester provides a description of ancient battle to introduce the idea that modern battle is a result of a concentration of resources versus a series of duels. The concentration of forces allows for a single target to be attacked from multiple sources and thus is no longer a simple duel. However, when it is assumed that the forces have equal individual fighting values, "the number of men knocked out per unit time will be directly proportional to the numerical strength of the opposing force [18]". Lanchester provides the following two equations, as mathematical representations of the preceding statement (Equation 4); when integrated, these are now referred to as the Lanchester Square Laws (Equation 5) [18].

$$\frac{dB}{dt} = -R \times c$$
$$\frac{dR}{dt} = -B \times k \tag{4}$$
$$k[R_0^2 - R^2(t)] = c[B_0^2 - B^2(t)] \tag{5}$$

Using the same assumptions from Taylor's Lanchester Linear Law example, Table 2 presents the impact of concentration of forces by solving the Lanchester Square Law.

Table 2: Modern Warfare Example (adapted from [19])

B_0	100	150	200	250	300
B_f	0	112	173	229	283
Blue losses	100	38	27	21	17

* Fighting continues until $R_f = 0$

The Lanchester Laws originally representing ancient and modern warfare, evolved into representing area and direct fire weapons. Fortunately, the analyst may not have to choose between the two laws and can instead use a combination. Helmbold expanded the Lanchester Laws by representing both the Linear and Square Laws in one general form given in Equation 6 [20].

$$\frac{dB}{dt} = c\left(\frac{B}{R}\right)^{1-w} R$$
$$\frac{dR}{dt} = k\left(\frac{R}{B}\right)^{1-w} B \tag{6}$$

When the Weiss parameter (w) equals 1, the Lanchester Square Law results. Similarly, when w = 1/2, the result is a form of the Lanchester Linear Law. Additionally, w = 0 describes the Lanchester Logarithmic Laws. The Lanchester Logarithmic Laws are typically used to describe the number of casualties of non-fighting participants, such as the doctors, chaplains, and headquarters staff [20].

Bracken introduced a tactical parameter to the Lanchester Laws in "Lanchester Models of the Ardennes Campaign". This parameter has been used by authors, since its introduction, to better fit the Lanchester Laws to historical battles (See Chen and Chu [21], Lucas and Turkes [22], and Hung *et al.* [23]).

Starting from the general form of the Lanchester Laws (Equation 6), Bracken added a tactical parameter, d, and rewrote w in terms of p and q, where p + q = 1, to result in Equation 7 [24].

$$\frac{dB}{dt} = c(d\ or\ 1/d)R^p B^q$$
$$\frac{dR}{dt} = k(1/d\ or\ d)B^p R^q \qquad (7)$$

The Red and Blue exponent parameters (p and q) are able to model the linear laws, square laws and the continuous range between. When $p = q = 0.5$ the linear laws result, and when $p = 1$ and $q = 0$ the square laws result.

Bracken's tactical parameter was used as a constant d for the defending force and 1/d for the attacking force. The parameter switched when the attacking force changed during the battle. When $d < 1$ the defender receives less casualties and when $d > 1$ the defender receives more casualties. Bracken, Chen and Chu, Lucas and Turkes, and Hung et al. all showed the data of the Ardennes Campaign (Battle of the Bulge) and the Battle of Kursk could be better fit using Bracken's Tactical Parameter, d. During both of these battles, the attacking force changed.

The Lanchester Laws were expanded by Deitchman [25] and Schaffer [26] to include tactics employed while engaged in guerrilla warfare. Deitchman published an early article applying the Lanchester Laws to insurgent warfare. This article, entitled "A Lanchester Model of Guerrilla Warfare," focused on the force ratios and size of the guerrilla bands versus the conventional/regular army. Deitchman suggested that an ambush set by the guerrillas was best described by the guerrillas attacking using direct fire, while the conventional army returned fire into an area or using indirect fire [25].

Schaffer, in his RAND research paper entitled Lanchester Models of Guerrilla Engagements [26], continued Deitchman's research. He specifically modeled three categories of guerrilla warfare; skirmishes, ambushes, and sieges. Schaffer began with a general form of the Lanchester Laws and added additional components to accurately model the characteristics of warfare. Similar to Deitchman, Schaffer assumed the ambushee initially engaged in area fire. However, as the battle continued, the ambushee transferred to aimed fire [26]. The ambushers employed aimed fire during the entire engagement. As will be demonstrated in the final example of this paper, system dynamics effortlessly represents this transition from area to direct fire. While this study's primary example is traditional attrition based warfare, system dynamics may be used to describe several elements of insurgency warfare.

5. Application

This study will compare and contrast the differences in modeling the Lanchester Square Law in both a discrete event simulation and system dynamics model. A simple case is used in this section to demonstrate how to apply the discussed framework. These illustrations can be solved analytically; however as the complexity of interactions and dimension of the state-space increase in an actual operational scenario, the analytic solution becomes prohibitive to solve. This is evident by the number of complex combat scenarios which require simulation to model the initial interactions and are therefore never reduced to analytic equations.

The Lanchester equations are implemented in discrete event simulations by solving the equations at discrete time intervals. When the analytic solution to the Lanchester Equations can not be found because of the complexity of the example, a discrete event simulation is not able to solve the equations to predict the *exact* time future events occurred. Therefore, the exact future event times are not generated. A discrete event simulation evaluates events at predetermined time intervals. This is referred to by Bartley as a synchronous discrete event simulation [27].

In general, synchronous discrete event simulation has several potential drawbacks. One drawback is non-simultaneous events must be treated as simultaneous if they fall within the same time period. This requires all possible material interaction combinations to be anticipated, modeled and planned. Additionally, as the step size is decreased, the accuracy of results increases; however, the speed of simulation decreases. To increase the speed of the simulation, it is possible to skip intervals if enough information about the system can be obtained. Skipping intervals must be based on expected conditions and the required fidelity of the solution, both further complicating the simulation [27].

A discrete event simulation must approximate the values of attributes and variables between events. This can be resolved by reducing event times, but it may force the model to take substantially more time to complete. A system dynamics model does not approximate to achieve the level between events. A benefit of system dynamics models, when required by the analysis, is their ability to smoothly transition the levels of the attributes and variables. This does not imply that the levels can not be adjusted in step increments as demonstrated later in this study with the implementation of reserve troops.

In some modeling scenarios the arrival of a regiment, army, or additional fighters can be adequately and appropriately modeled as a discrete event, perhaps as the average time. In other settings, the actual flow of troops into the fight may be critical, suggesting the use of system dynamics. An example might be the arrival of Marschall Blücher's forces at Waterloo. While the main body had not totally arrived, the appearance of the forward elements on the battlefield was instrumental in turning the tide of battle.

There are numerous methods to model the Lanchester Laws. This study implemented the basic Lanchester Laws in a simple discrete event simulation. The entities generated within the discrete event simulation, for this study, do not have any attributes, nor require any resources. As real world aspects (like ammunition, equipment and supplies) are added to the model, they would be modeled as attributes or resources. Figure 2 is the pictorial representation of a basic implementation of the Lanchester Square Law developed in Arena 10.0.

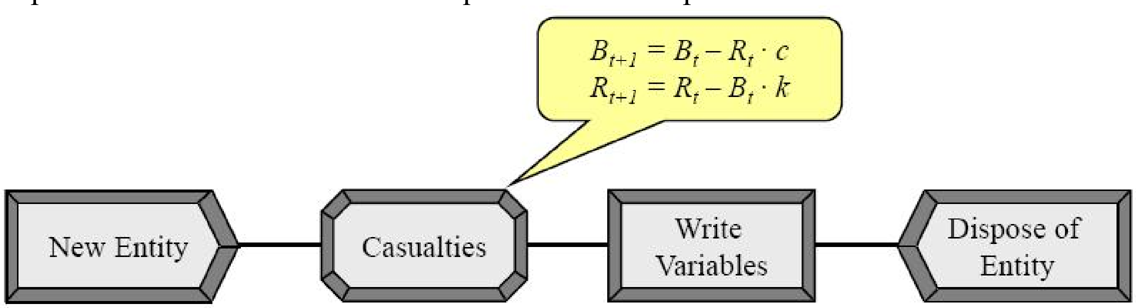

Figure 2: Discrete Event Simulation of the Lanchester Square Law

The furthest left container created a new entity at each time interval. The entity flowed to the Casualties container where the global variables representing the number of Blue and Red forces at time t + 1 (Bt+1 and Rt+1) were calculated based on the attrition coefficients (c and k) and the current size of the forces (Bt and Rt). After the global variables were adjusted, the simulation proceeded to write the status of the variables and then the entity was disposed. The balloons on the figure display the equation within each container.

The system dynamics representation focused on the level of Blue and Red Forces (Bt and Rt), where the forces are the number of personnel or the total combat value. Blue and Red force were changed by the rate of Blue or Red Attrition. Figure 3 is the depiction of the system dynamics representation of the basic illustration, developed using the software package Vensim. Red Attrition through time (RAt) was the total number of Red Forces killed (or otherwise removed from battle) by the surviving Blue Forces during each time interval, and vice versa for Blue Attrition (BAt). The Blue Attrition Coefficient (k) represented the number of force of Red that could be killed per unit of Blue force.

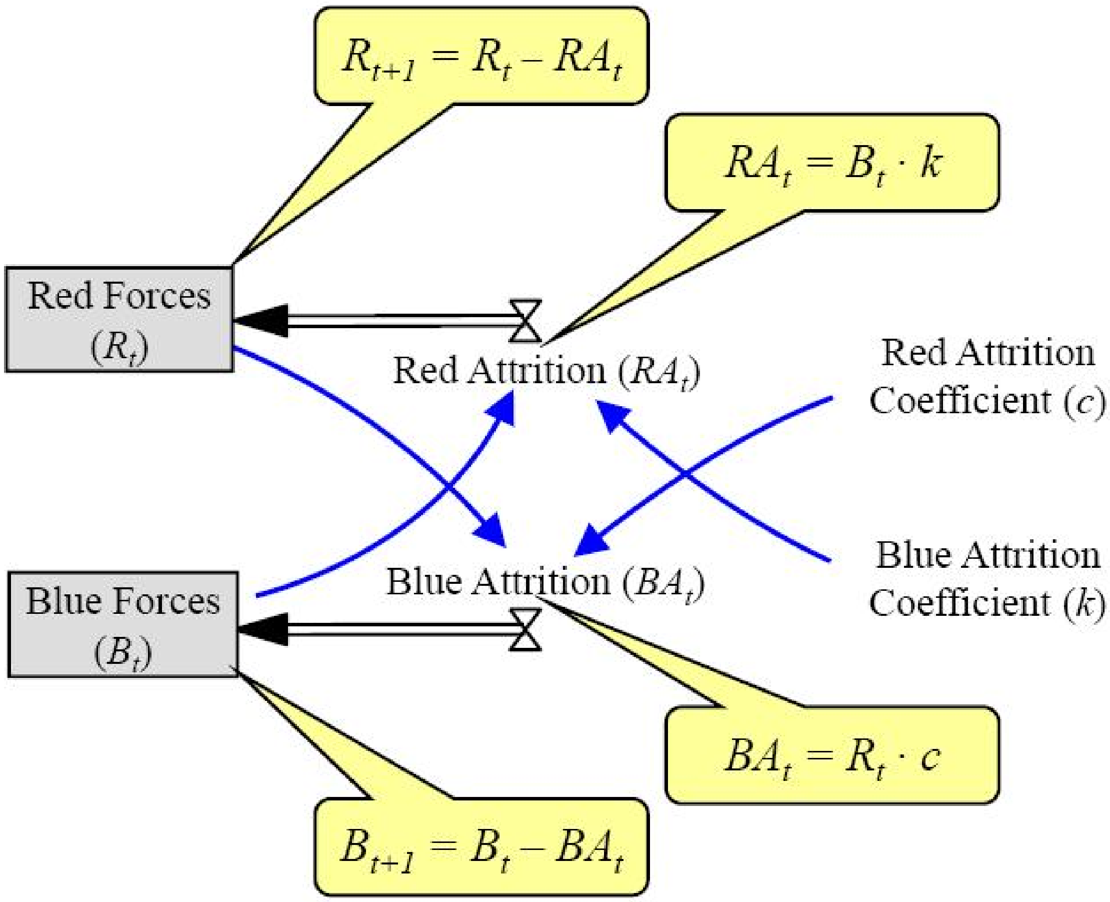

Figure 3: System Dynamics Model of the Lanchester Square Law

The basic discrete event simulation and system dynamics models were modified to represent Dewar, Gillogly, and Juncosa's simple combat model presented in Table 3 [28]. Dewar, Gillogly, and Juncosa simulated a two sided conflict with troop replacements using a discrete event simulation with a time-step of one-half hour. Their model used the Lanchester Square Laws with a Red Attrition Coefficient (c) of 1/2048 and the Blue Attrition Coefficient (k) that equaled 1/512. The initial Red (R_0) and Blue (B_0) troop sizes are also presented in Table 3. When specific thresholds based on a force ratio or percent of surviving forces were breached, reinforcements were requested. The reinforcement block size was 300 troops. Each side had five reinforcement blocks and the reinforcements arrived 70 time steps (or 35 hours) after the reinforcement threshold were breached. Withdrawal (or defeat) of the forces was declared once additional predetermined thresholds for either combat ratio or percent of surviving forces was crossed. Victory was declared based on a specific combat ratio (i.e. forces were substantially outnumbered), or a force strength dropping below a predetermined level (i.e. the battle was too costly to continue) based on the thresholds in Table 3.

Table 3: Simple Combat Model (adapted from [28])

Parameter	Blue	Red
Initial troop strength	$B_0 = 839$	$R_0 = 1500$
Combat attrition calculation	$B_{t+1} = B_t - R_t/2048$	$R_{t+1} = R_t - B_t/512$
Reinforcement threshold	$R_t/Bt \geq 4$ or $B_t < 0.8\ B_0$	$R_t/B_t \leq 2.5$ or $R_t < 0.8\ R_0$
Withdrawal threshold	$R_t/B_t \geq 10$ or $B_t < 0.7\ \mathrm{B}_0$	$R_t/B_t \leq 1.5$ or $R_t < 0.7\ R_0$

Figure 4 depicts the discrete event simulation which represents Dewar, Gillogly, and Juncosa's combat model. The Casualties were computed as described for Figure 2. Next, the model checked to see if the conditions required to declare the winner were met. This is represented in Figure 4 as the diamond labeled 'winner.' If there was a winner, the global variable indicating a combat force had won was changed. If there was not a winner, the model checked to see if the Blue side was currently waiting for reserves. If the Blue side was not waiting for reserves, the model checked the Blue reinforcement thresholds to see if Blue Reinforcements were necessary. If reserves were required, the global variable for Blue Waiting for Reserves was triggered, and the model scheduled the event to add reinforcements in 70 time steps. The model reset the Blue Waiting for Reserves variable after 70 time steps and disposed of the entity. If the system was waiting on Blue Reserves or did not need Blue Reserves, similar checks were made regarding Red Reserves. This implementation did not allow for both Red and Blue forces to add Reserves at the same time period. This was possible to implement but would add an additional layer of condition checks and directives to the model. As discussed earlier, a drawback of synchronous discrete event simulations is all possible material critical state-spaces and conditions must be anticipated and those paths and conditions must be tested and implemented within the model.

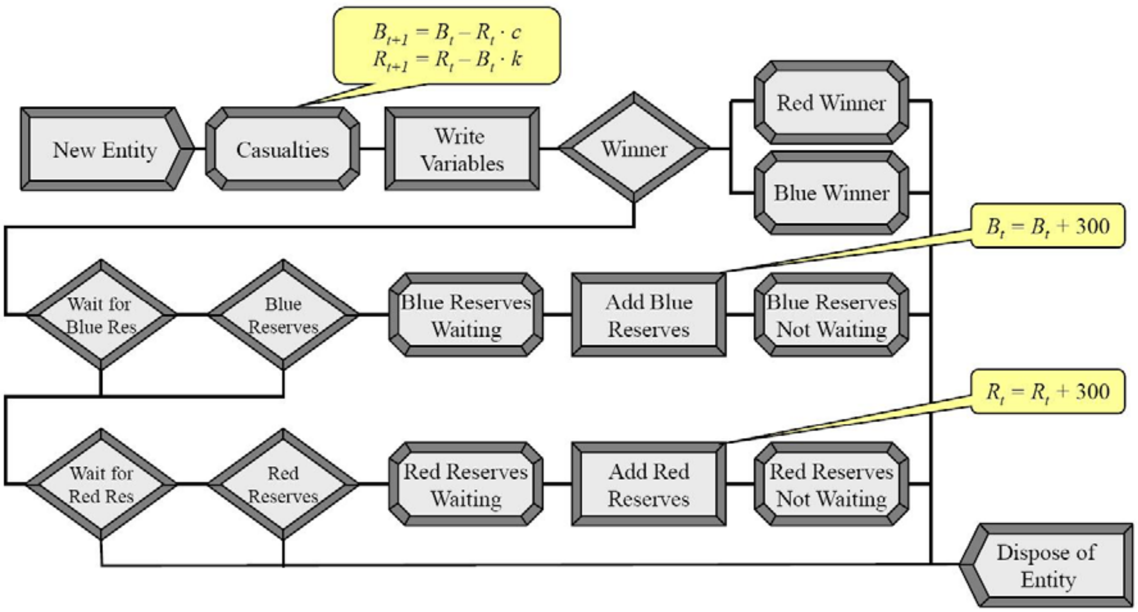

Figure 4: Discrete Event Simulation of the Simple Combat Model

The results of the discrete event simulation are presented in Figure 5. The discontinuous points are due to the arrival of the 300 reinforcements. The Red Forces required all of their reserves, while the Blue Force won after only adding three reserve blocks.

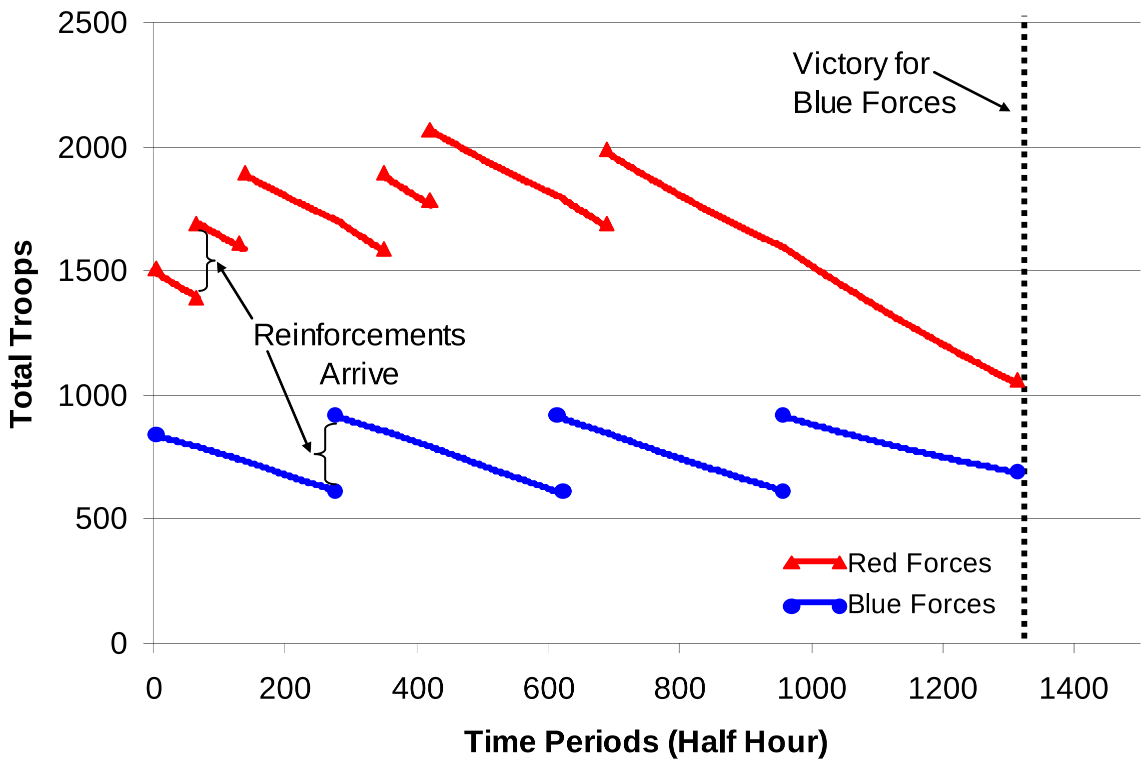

Figure 5: Results of the Combat Model

The basic system dynamics model was modified to represent Dewar, Gillogly, and Juncosa's combat model by first creating the levels of Blue/Red Force Strength and Combat Ratio. The Blue/Red Force Strength was the percent of forces still alive at the current interval of the battle. The Combat Ratio was the Red Forces divided by the Blue Forces. The Combat Ratio and Blue/Red Force Strength levels were required to determine the requirement of Red and Blue Reserves. If the reserve force thresholds were achieved, the system checked to see if it was waiting on reserves and if there were reserve blocks available. If all conditions were met to add the new reserves, the value labeled Add Red Reserves (RRt) and/or Add Blue Reserves (BRt) was changed to 300 and the rate flowed to the Red and Blue Forces respectively.

The system dynamics representation of the simple combat model is presented in Figure 6. As opposed to the discrete event simulation, the system dynamics model allowed both Red and Blue Forces to be added at the same time, since the addition of troops does not require all possible interaction combinations to be anticipated for each time interval. Additionally, the time the reserves were requested could be identified at the exact moment the thresholds were reached. The timing could be more precisely identified within the discrete event simulation by reducing the time step, but the time step was already small relative to the scenario.

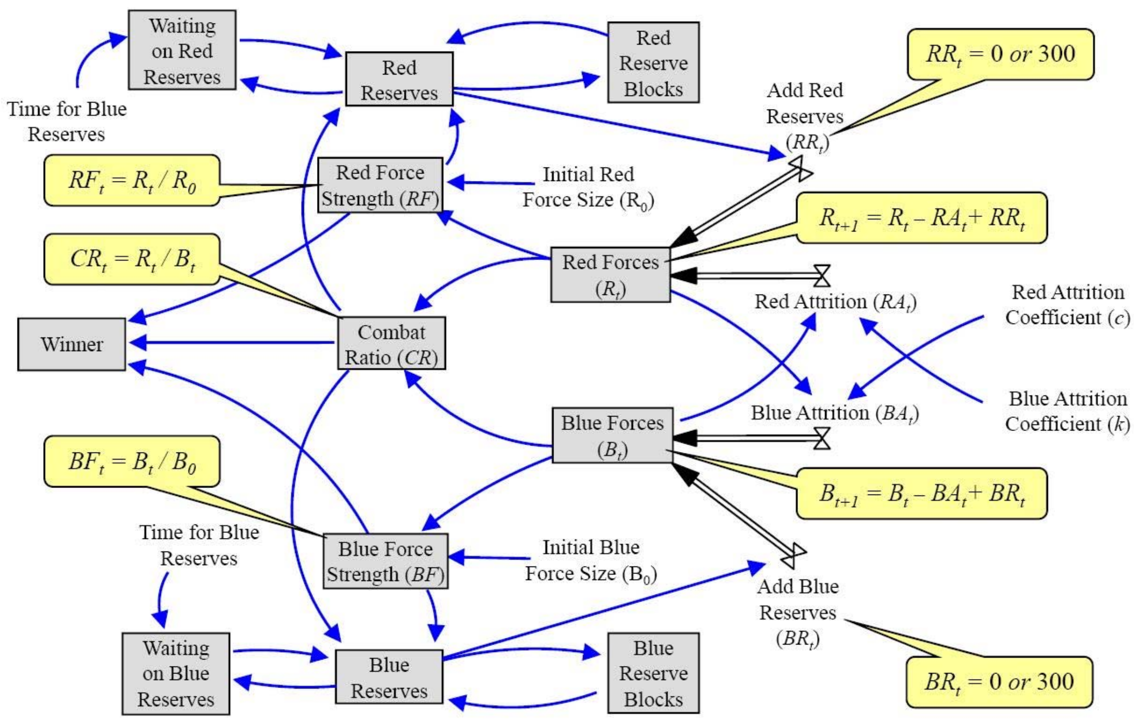

Figure 6: System Dynamics Model of the Simple Combat Model

The discrete event simulation and system dynamic models produce equivalent results. In both representations of the model, the reserves arrive on the same day and the battle concluded on the same day. There were only minor variations in the force size throughout the run of the model. The largest difference between the sizes of forces reported by each model for the 1,310 time intervals was 0.0013 or 0.00011% of the response. Figure 5 can be used to represent the results from both the discrete event simulation and the system dynamics models. The correlation of Blue force at the end of each time interval of the discrete event simulation with results from the system dynamics model was 1 – 1.29E-12. Red correlation was 1 – 1.11E-12. Both represent a correlation of results which is approximately one. This exercise demonstrates that system dynamics models can effectively model the Lanchester Laws which have been historically modeled as a discrete event simulation; in addition if a continuous flow more accurately models the environment, a system dynamics model may be preferred.

The system dynamics model of these events and rates can be easily adjusted based on the current environment. For example, if the reserves were located near the engaged troops, the time of arrival of the reserves could be less than when the front line of troops were at a distance. Additionally, the system dynamics models could 'flow' the reserves into the engaged forces as logistics allowed rather than assuming all the reserves arrived at the same time (as typically expected within a discrete event simulation). A system dynamics based model can easily incorporate this change by connecting the 'level' indicating the troop's location to the 'level' indicating time to wait for reserves. The time to wait on the reserves is no longer a constant and can depend on the location of troops and the rate of their arrival. In addition, the rate of arrival of troops to the front can be controlled, if desired.

The discrete event simulation and system dynamics examples focused on the engagement, or battle, level of warfare. As the Lanchester Laws have been applied to all levels of warfare (engagement, mission, campaign and strategic), both approaches can be used as a modeling framework for all levels of warfare. This is not meant to suggest current discrete event simulations are inadequate or inappropriate. Rather as combat and conflict become more affected by continuous, dynamic interacting elements, especially at the campaign or theater level and the data to model such events becomes more available, system dynamics may offer a different level of insight to decision makers. Historically, system dynamics has been applied to a vast array of levels in planning from the factory floor to world models (seen in [7] to [12], among others). This suggests that it can be applied from skirmishes through strategic levels, given proper inputs. System dynamics models are particularly appropriate when not accounting for the continuation of time, leading to issues of accuracy. Its consideration provides another modeling option. As with all modeling, the analyst should select the appropriate technique for the fidelity required by the decision environment.

6. Implementation of Fatigue

Thus far the values of the attrition coefficients have been static. While in real combat the troops would become more or less effective as the battle continues and fatigue, among other attributes, change. As a demonstration of potential options, the dynamic attributes of the system dynamics model was expanded by incorporating a 'fatigue' coefficient into the Dewar, Gillogly, and Juncosa's combat model. To model this, fatigue was represented as the effective combat power for a force. A force having no fatigue retains 100% of its combat power. Over time while in combat, fatigue increases, effectively reducing combat power. The new attrition coefficient now equaled the original coefficient multiplied by one minus the fatigue factor. It was first assumed that the initial forces have no fatigue. It was further assumed that the reserves entered the conflict with no fatigue. When the reserves entered, the percentage of fatigue was reset to reflect the addition of fresh troops.

In this excursion it was assumed that when the reserves entered, the new fatigue was a weighted average of the fatigue from the reserves and the existing forces. For example, when 1000 troops were fatigued at 30% and 500 fresh reserves entered combat, the new fatigue value was two thirds (1000/1500) of its previous value or 20% based on this illustrative fatigue aggregation formula (any appropriately vetted factor may be used). This implementation implied that the reserves were mixed throughout the current combatants when the reserves entered combat (of course, a damping function or morale boost could be included).

Fatigue is modeled in the discrete event simulation by the creation of four new global variables. These could be referred to as the Blue/ Red Attrition Coefficients (c / k) and Blue/ Red Fatigue (BF / RF). Additionally, new decision nodes are required to decide how, why and when to change these new global variables. The alterations are possible, but not insignificant.

The system dynamics model added two new levels (identified as Red and Blue Fatigue). The system dynamics model with the fatigue factor is presented as Figure 7. Red and Blue Fatigue were a function of the rate reserves entered the system and the current level of forces. The attrition coefficients (c and k) were now a function of fatigue.

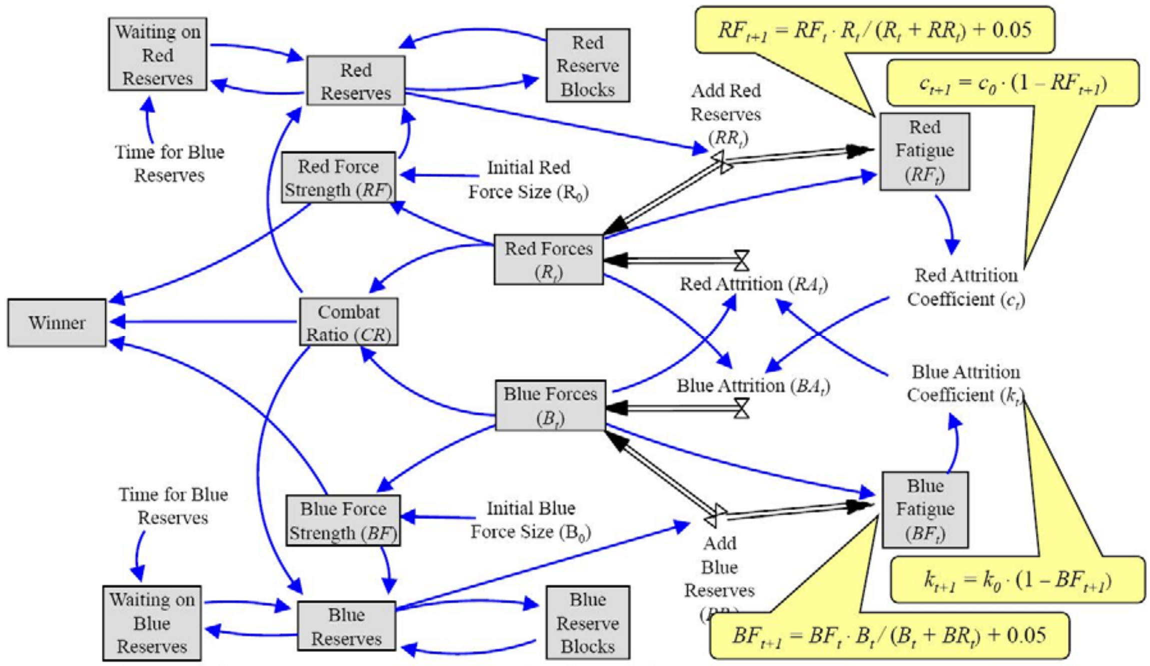

Figure 7: System Dynamics Model with Fatigue

The systems dynamics based modeling approach did not require the effect be modeled as a new fatigue level; however, by directly representing this new term, the model emphasized the new factor. The concept of fatigue could have been directly represented within the attrition coefficient value with the arrows from 'Red Forces' and 'Add Red Reserves' being connected to the appropriate 'Attrition Coefficient.' Figure 8 presents the fatigue of each force over time.

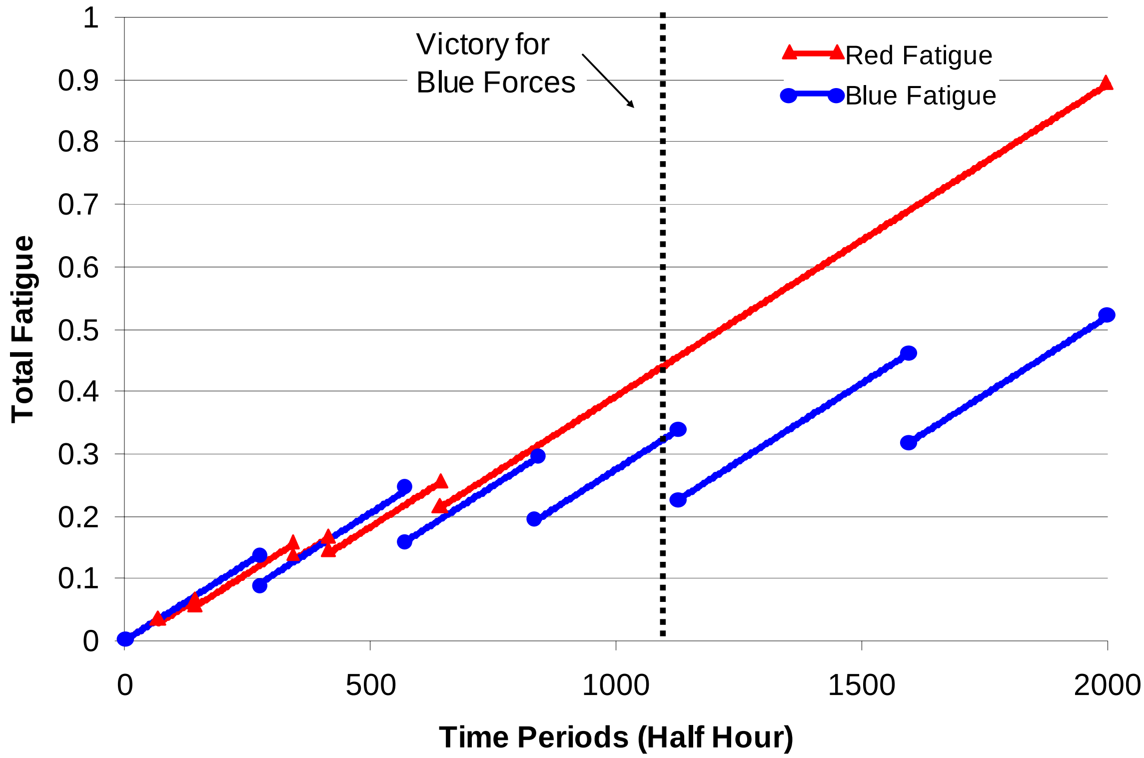

Figure 8: Fatigue of Red and Blue Forces

The arrival of Red reserves through 300 hours of conflict had the effect of reducing overall Red fatigue. Once all the reserves were committed, Red fatigue impacted Red's overall effectiveness since relief was not available later in the battle. This was opposed to the Blue forces which maintained less battlefield fatigue. The model was run beyond the Blue victory termination point to demonstrate this point.

Figure 9 compares the force levels of both the Red and Blue forces with and without the effects of fatigue. Not only did both sides suffer more losses, the battle ended significantly sooner due to the impacts of fatigue. While fatigue was added in this simple example, any number of factors can be incorporated as appropriate. System dynamics offers an additional option to the modeler when the scenario and fidelity of the analysis requires this level of detail.

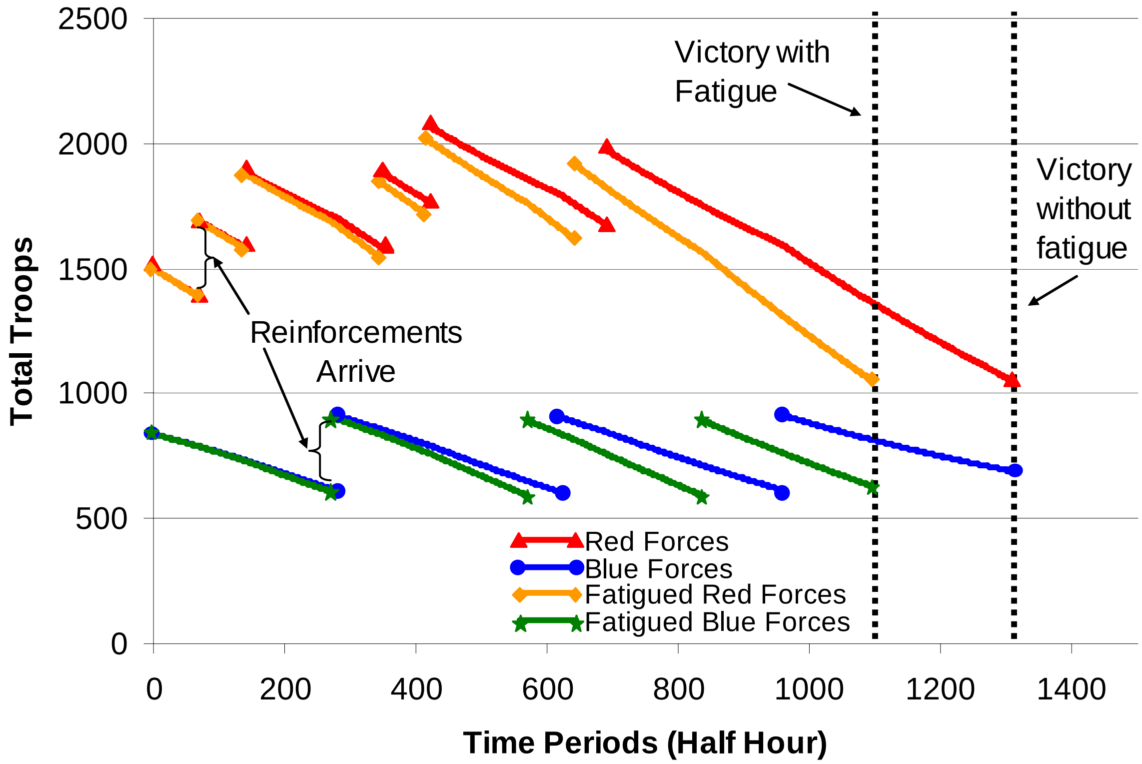

Figure 9: Comparison of Results

This example demonstrated how to dynamically represent the Lanchester Laws within a dynamic conflict model and shows the inclusion of a softer factor. By including fatigue, a qualitative aspect has been directly incorporated into the combat effectiveness coefficient. The rate of attrition is now altered by the change of fatigue. Therefore, there is now a change of rates being included in the combat model. The combat ratio and attrition is still used by the system dynamics model; however, the values of the combat effectiveness coefficients are no longer static. The coefficients are now dynamic and continuous. This changes the paradigm generally used in discrete event simulation which focuses on the events in the model. System dynamics highlights the rates of change. While Lanchester Laws requires 'breakpoints' to declare victory, the focus is on the rate of attrition. System dynamics provides an ability to change the rates based on other, possibly softer, aspects. This indicates a strength provided by dynamically representing the Lanchester Laws if appropriate to the analysis.

Figure 10 provides a campaign level example of how the change of rates can be impacted by softer aspects. This figure represents the attrition of the Red Forces as Helmbold's general form of the Lanchester Equations (Equation 6).

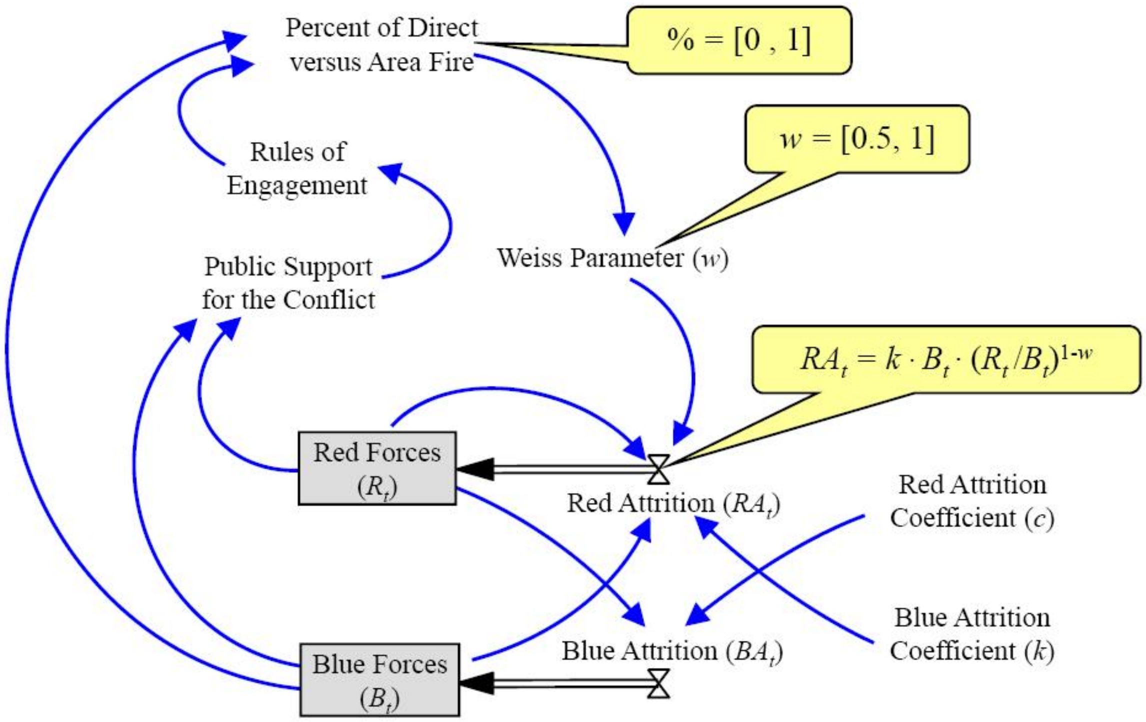

Figure 10: Dynamic Model of Hembold's General Form of the Lanchester Laws

Figure 10 indicates that Public Support for the Conflict is influenced by the number of Red and Blue Forces fighting and effects the Rules of Engagement. The Rules of Engagement dictate Blue's response when attacked. The response is the fighting behavior of the Blue Forces. When the rules require 100% positive confirmation of the target, Blue's response may be entirely direct fire, which sets the Weiss Parameter at 1 (resulting in the Lanchester Square Laws). However, when collateral damage is acceptable, the Rules of Engagement may allow an Area Fire approach; resulting in the Weiss Parameter set at 0.5 and consequently the Lanchester Linear Law. Since the model is dynamic, the value of the Weiss Parameter may continuously vary between these extremes to appropriately model the engagements at a campaign level.

This simple example illustrates how feedback loops can be used in a system dynamics simulation. While 'softer' factors were used, the approach can be applied to any factor where rates of continuous flow are to be modeled. Although this example is at the campaign level of warfare, without difficulty it can be modified to represent the guerrilla tactics of skirmishes, ambushes, and sieges proposed by Schaffer.

7. Conclusion

Discrete event simulation and system dynamics models have been used to approximate first and second generation warfare, also known as attrition warfare. The Lanchester Laws are a substantiation of attrition modeling. Additionally, third generation warfare or maneuver warfare can be represented by both frameworks. However, in some operational settings it may be insufficient to model only military attrition. This can be particularly true in insurgency warfare or the broader fourth generation warfare or operations. All instruments of power (diplomatic, informational, military, and economic) must be represented. System dynamics provides an ability to include the impacts of the softer aspects of diplomacy, information, economics, as well as military actions, when continuous flows might provide improved insight over discrete event simulation. Feedback loops and changing the rate of rates can be useful in capturing the broader non-attrition and non-maneuver aspects of conflict.

This paper compares the use of discrete event simulation and system dynamics to model first and second generation warfare (example on Lanchester Laws) to establish the viability of the approach. The paper then provided a simple system dynamics model which fatigued the forces. The fatigued forces provided an example of how other aspects required when modeling insurgency warfare or fourth generation operations could be implemented into combat models.

A systems dynamics presentation was developed to model a two sided conflict subject to continuous, non-linear, transient behaviors with feedback. The system dynamics presentation of the model provides the decision maker with a clearer representation of a continuous system than a discrete event simulation. The specific interactions are represented as levels, rates, and decision functions. The system dynamics format can be more straightforward than the sometimes ‘black box’ appearing presentation of a discrete event simulation.

System dynamics models can be quickly adjusted to represent new assumptions. Additionally, as discussed in the introduction to system dynamics, system dynamics models are able to represent ‘softer’ concepts that are traditionally not approached by discrete event modelers. Furthermore, system dynamics models smoothly transition the values within the levels, as opposed to approximating the values as required by a discrete event simulation.

While some discrete event simulations are able to represent continuous variables, this does not imply the simulation represents completely dynamic models. When incorrect conclusions could occur due to not accounting for continuous time, a model developed to implement a system dynamics framework should be used. However, if a mix of discrete and continuous variables is advantageous in a modeling effort, systems which incorporate both can be developed and utilized. Care in selecting time steps must be taken to assure accuracy sufficient for the analysis.

The implementation of Lancaster Laws within a system dynamics model adds a significant amount of flexibility to how combat models are able to incorporate the dynamics of battle. This added flexibility provides future research a means to model the impacts of the population’s resolve and combatants’ spirit. Additionally, this may allow combat models to capture many other aspects of the conflict such as the political, economic, and informational; rather than exclusively the military impacts.

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